CS448f: Image Processing For Photography and Vision

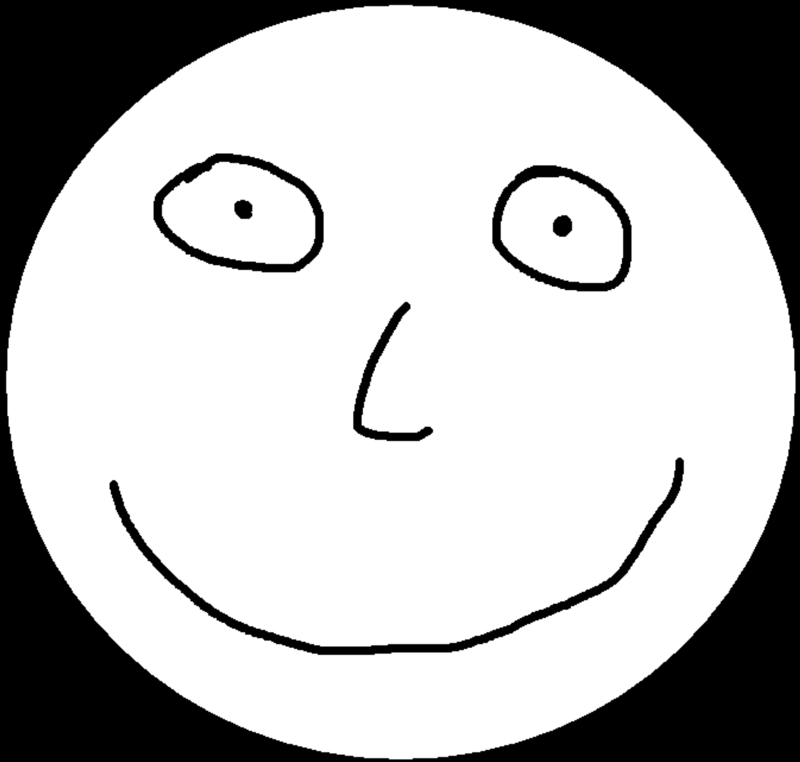
Fast Filtering Continued

Filtering by Resampling

- This looks like we just zoomed a small image

- Can we filter by downsampling then upsampling?

Filtering by Resampling



Filtering by Resampling

- Downsampled with rect (averaging down)
- Upsampled with linear interpolation

Use better upsampling?

- Downsampled with rect (averaging down)
- Upsampled with bicubic interpolation

Use better downsampling?

- Downsampled with tent filter
- Upsampled with linear interpolation

Use better downsampling?

- Downsampled with bicubic filter
- Upsampled with linear interpolation

Resampling Simulation

Best Resampling

- Downsampled, blurred, then upsampled with bicubic filter

Best Resampling

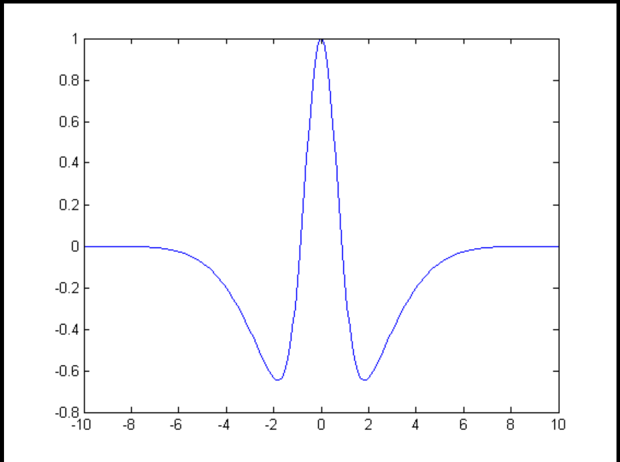
- Equivalent to downsampled, then upsampled with a blurred bicubic filter

What's the point?

- Q: If we can blur quickly without resampling, why bother resampling?
- A: Memory use
- Store the blurred image at low res, sample it at higher res as needed.

Recap: Fast Linear Filters

1) Separate into a sequence of simpler filters
   - e.g. Gaussian is separable across dimension
   - and can be decomposed into rect filters
2) Separate into a sum of simpler filters



Recap: Fast Linear Filters

3) Separate into a sum of easy-to-precompute components (integral images)

   - great if you need to compute lots of different filters

4) Resample

   - great if you need to save memory

5) Use feedback loops (IIR filters)

   - great, but hard to change the std.dev of your filter

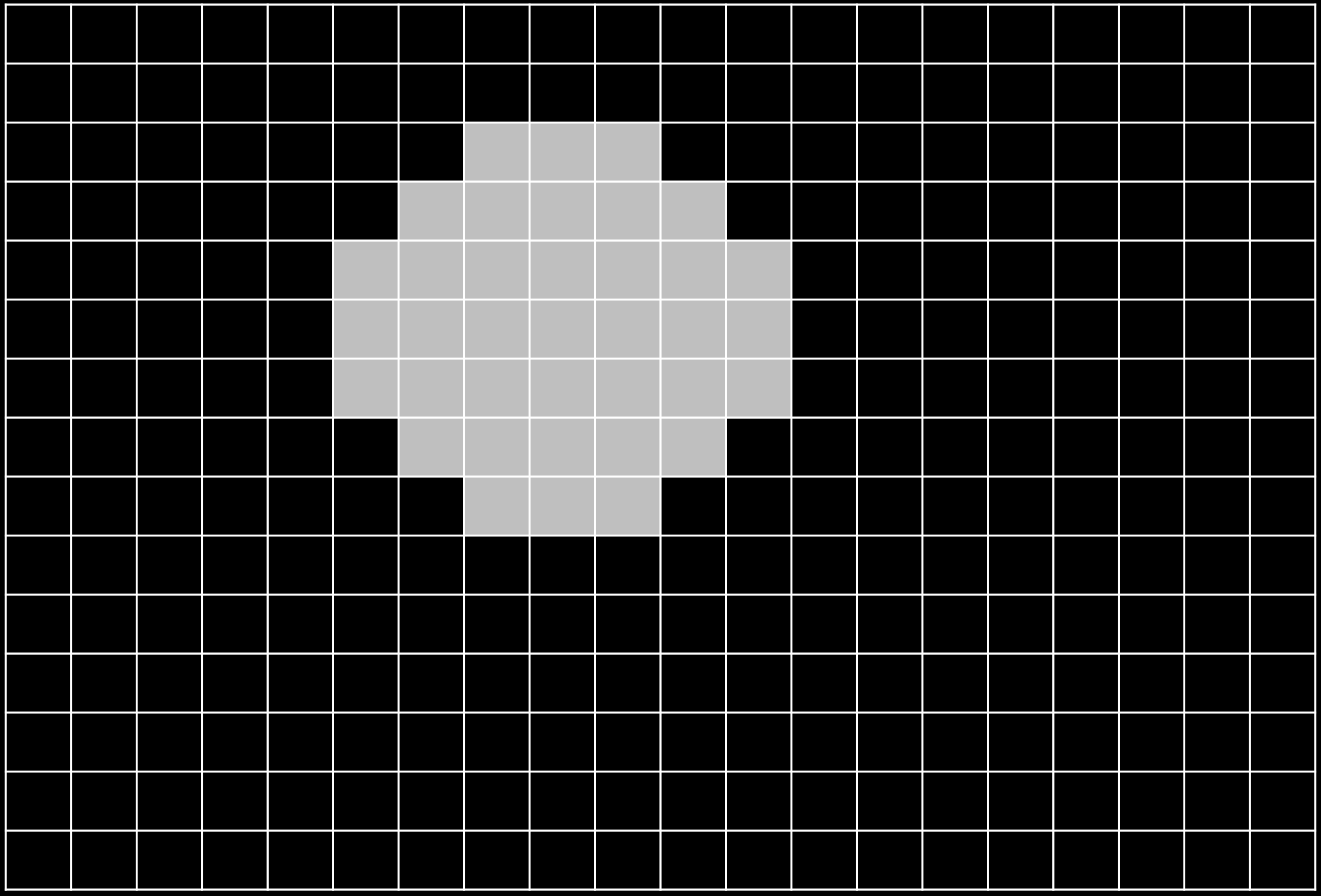
Histogram Filtering

- The fast rect filter
  - maintained a sum
  - updated it for each new pixel
  - didn't recompute from scratch
- What other data structures might we maintain and update for more complex filters?

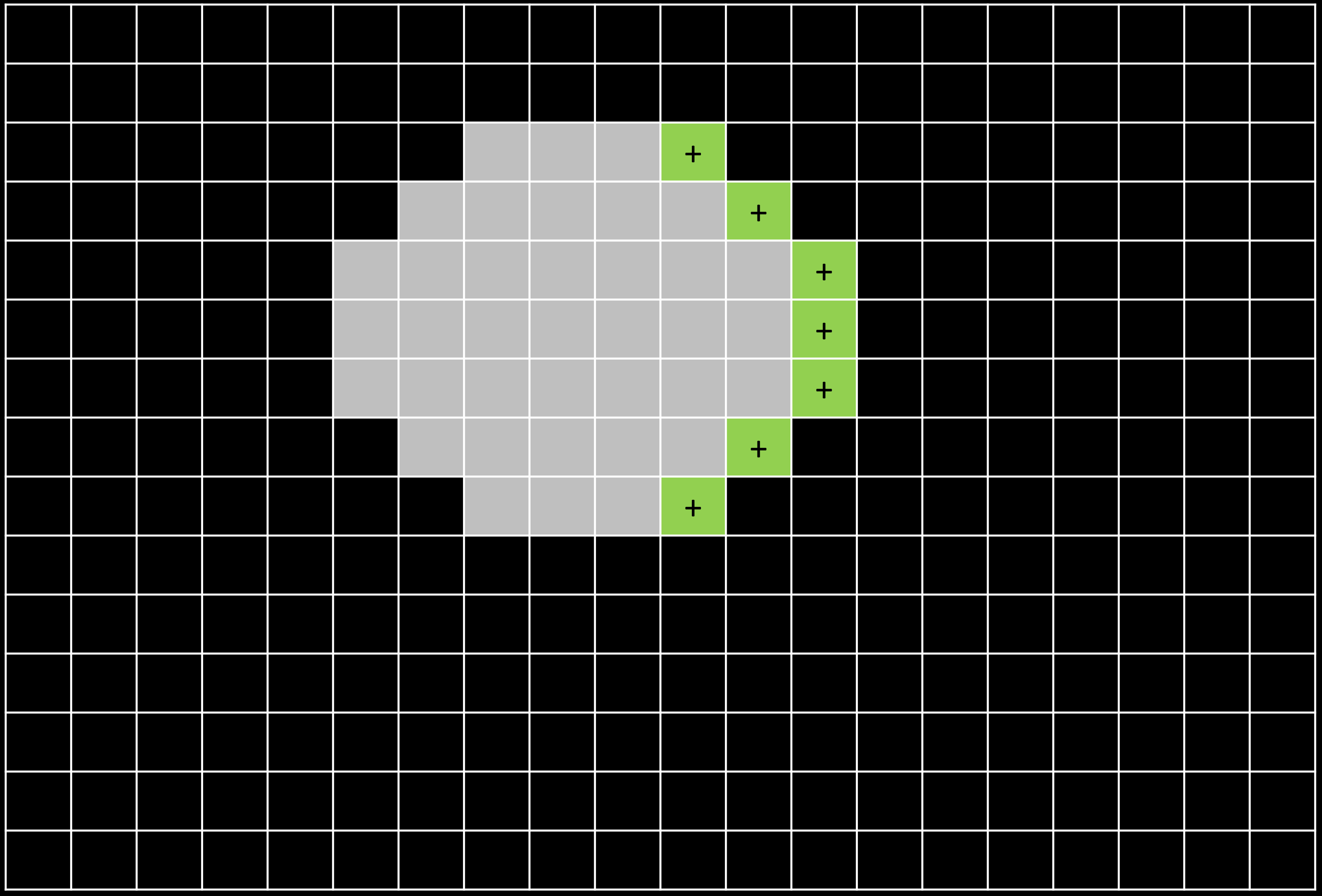
Histogram Filtering

- The min filter, max filter, and median filter
  - Only care about what pixel values fall into neighbourhood, not their location
  - Maintain a histogram of the pixels under the filter window, update it as pixels enter and leave

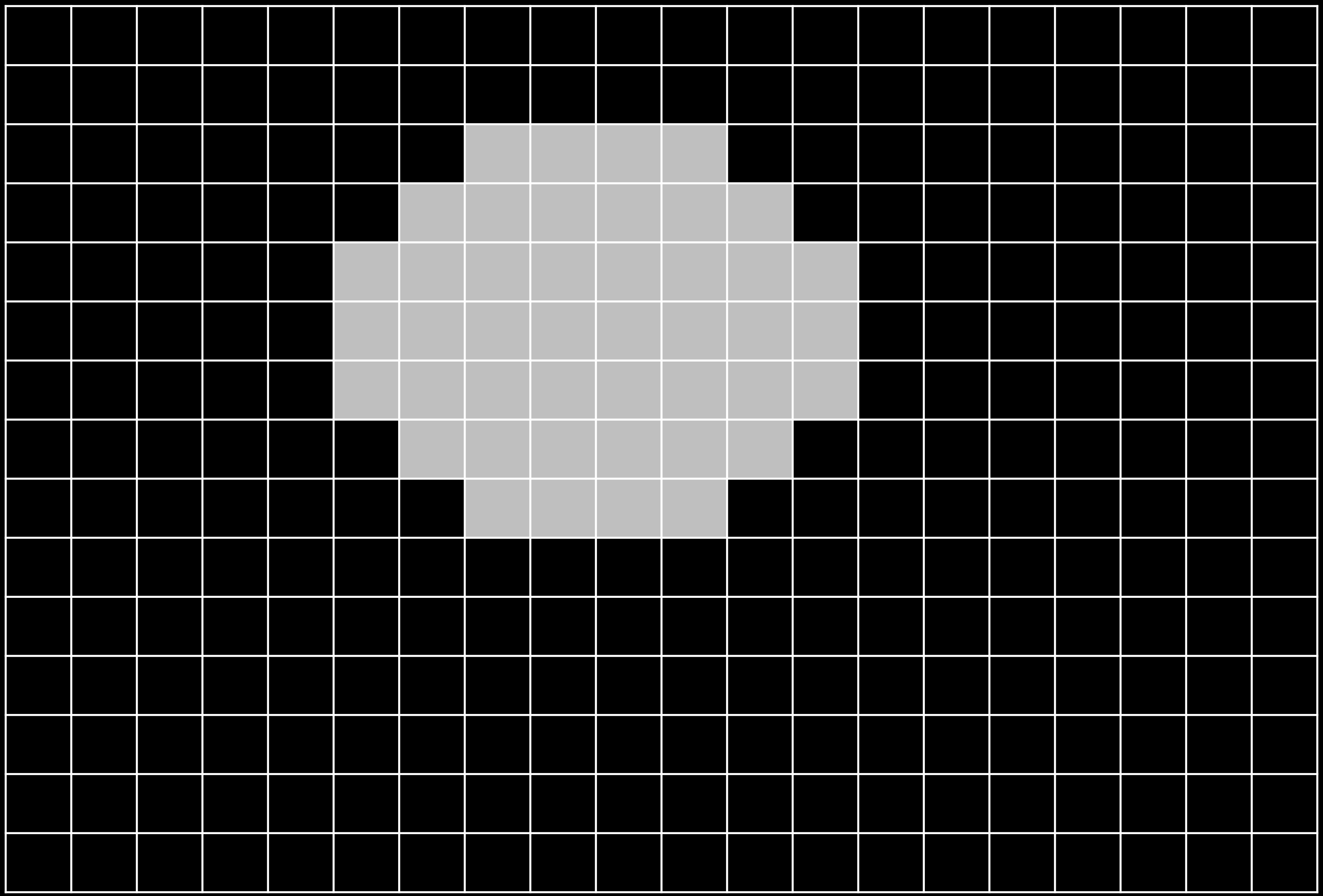
Histogram Updating



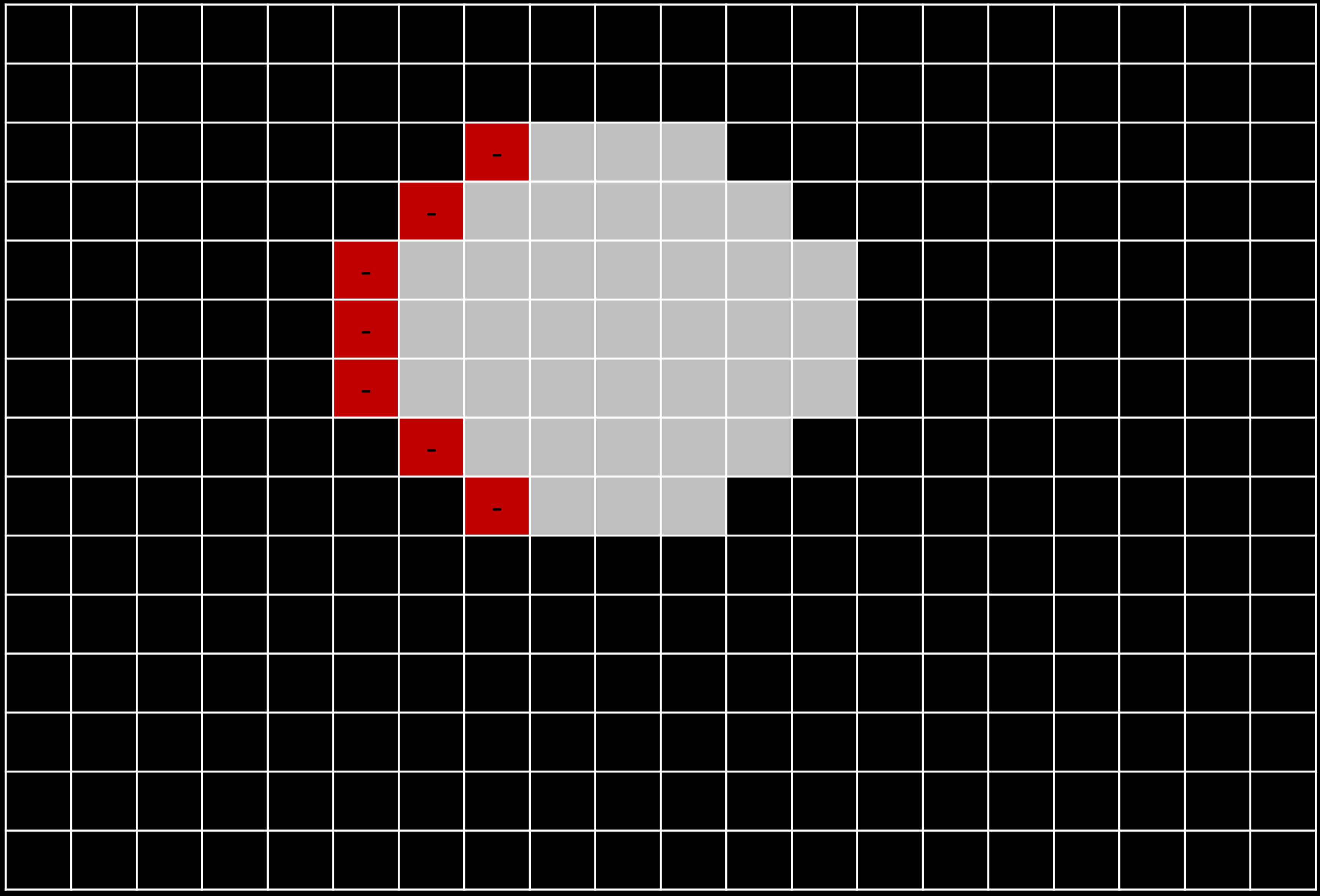
Histogram Updating



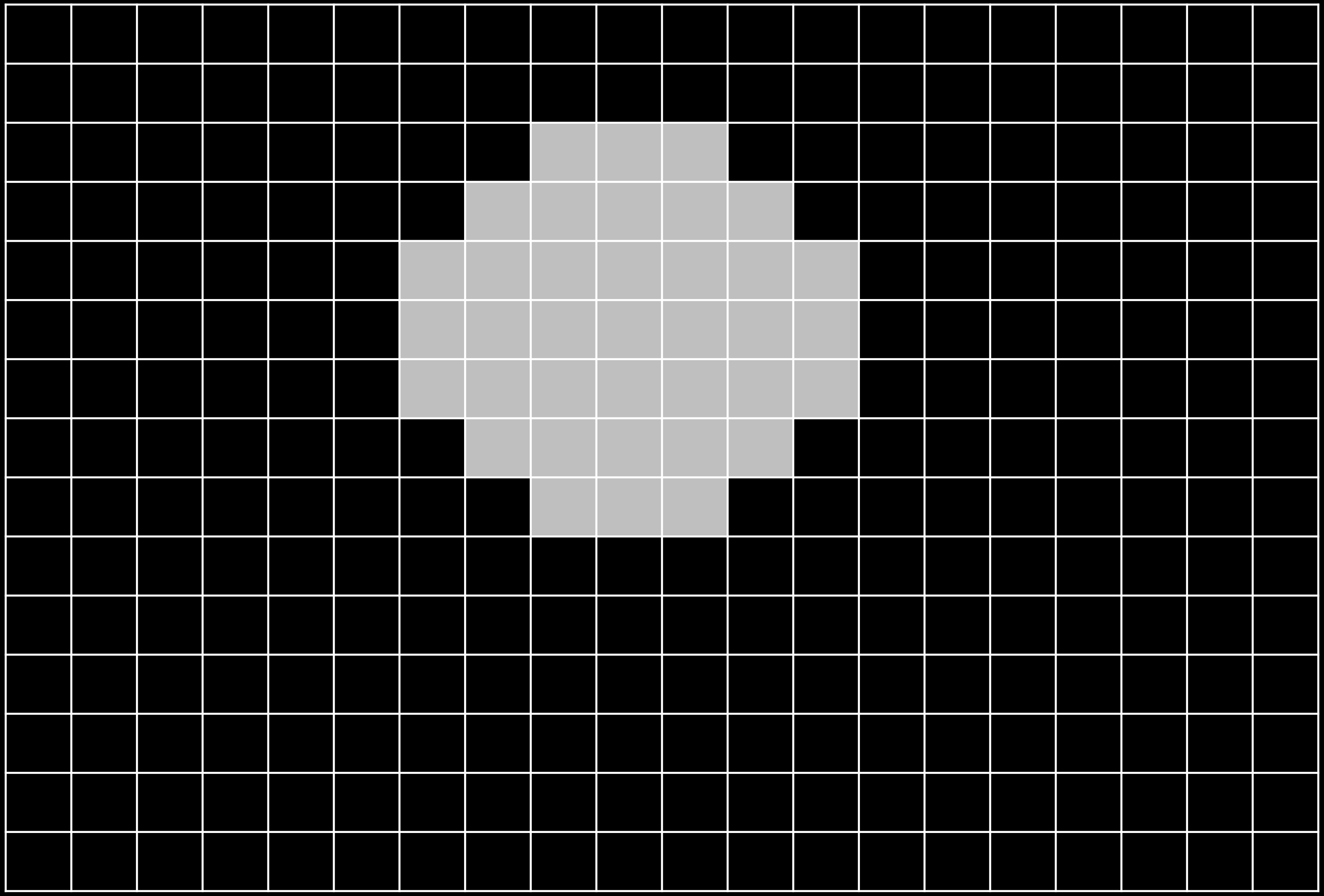
Histogram Updating



Histogram Updating



Histogram Updating



Histogram-Based Fast Median

- Maintain:
  - hist = Local histogram
  - med = Current Median
  - lt = Number of pixels less than current median
  - gt = Number of pixels greater than current median

Histogram-Based Fast Median

- while (lt < gt):
  - med--
  - Update lt and gt using hist
- while (gt < lt):
  - med++
  - Updated lt and gt using hist

Histogram-Based Fast Median

- Complexity?
- Extend this to percentile filters?
- Max filters? Min filters?

Use of a min filter: dehazing

Large min filter

Difference (brightened)

Weighted Blurs

- Perform a Gaussian Blur weighted by some mask
- Pixels with low weight do not contribute to their neighbors
- Pixels with high weight do contribute to their neighbors

Weighted Blurs

- Can be expressed as:

$$O(x) = \frac{\sum_{x'=x-f}^{x+f} I(x').e^{-(\sigma_1 (I(x)-I(x'))^2)}.w(x')}{\sum_{x'=x-f}^{x+f} e^{-(\sigma_1 (I(x)-I(x'))^2)}.w(x')}$$

- Where w is some weight term
- How can we implement this quickly?

Weighted Blurs

- Use homogeneous coordinates for color!
- Homogeneous coordinates uses (d+1) values to represent d-dimensional space
- All values of the form [a.r, a.g, a.b, a] are equivalent, regardless of a.
- To convert back to regular coordinates, divide through by the last coordinate

Weighted Blurs

- This is red: [1, 0, 0, 1]
- This is the same red: [37.3, 0, 0, 37.3]
- This is dark cyan: [0, 3, 3, 6]
- This is undefined: [0, 0, 0, 0]
- This is infinite: [1, 5, 2, 0]

Weighted Blurs

- Addition of homogeneous coordinates is *weighted averaging*
- $[x.r_0\ x.g_0\ x.b_0\ x] + [y.r_1\ y.g_1\ y.b_1\ y]$

$= [x.r_0+y.r_1 \quad x.g_0+y.g_1 \quad x.b_0+y.b_1 \quad x+y]$

$= [(x.r_0+y.r_1)/(x+y)$

$(x.g_0+y.g_1)/(x+y)$

$(x.b_0+y.b_1)/(x+y)]$

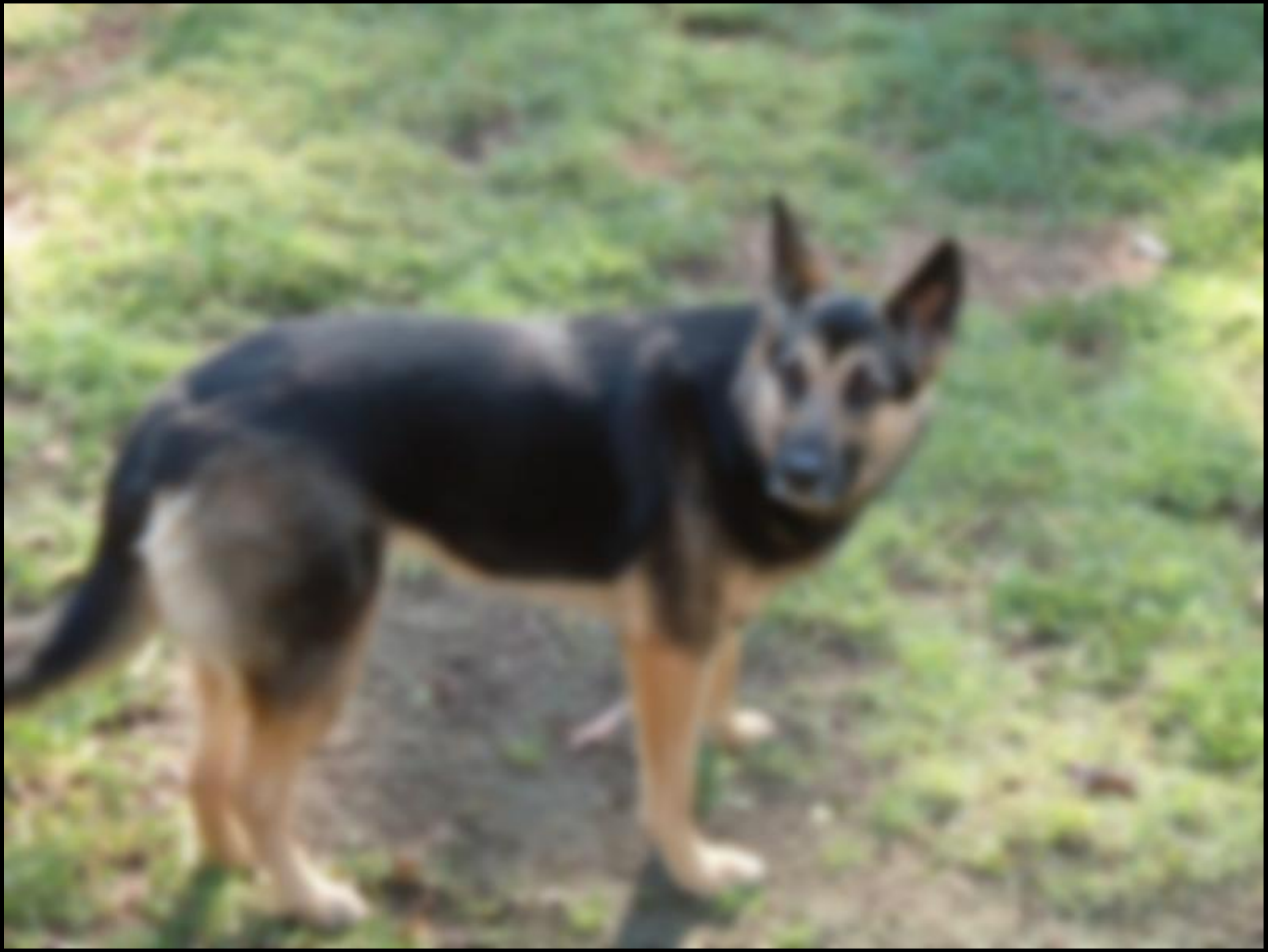
Weighted Blurs

- Often the weight is called alpha and used to encode transparency, in which case this is known as “premultiplied alpha”.
- We’ll use it to perform weighted blurs.

Image:

Weight:

Result:



Result:

- Why bother with uniform weights?
- Well... at least it gets rid of the sum of the weights term in the denominator of all of these equations:

$$O(x) = \sum_{x'=x-f}^{x+f} I(x').e^{-(\sigma_1 (I(x)-I(x'))^2)}$$

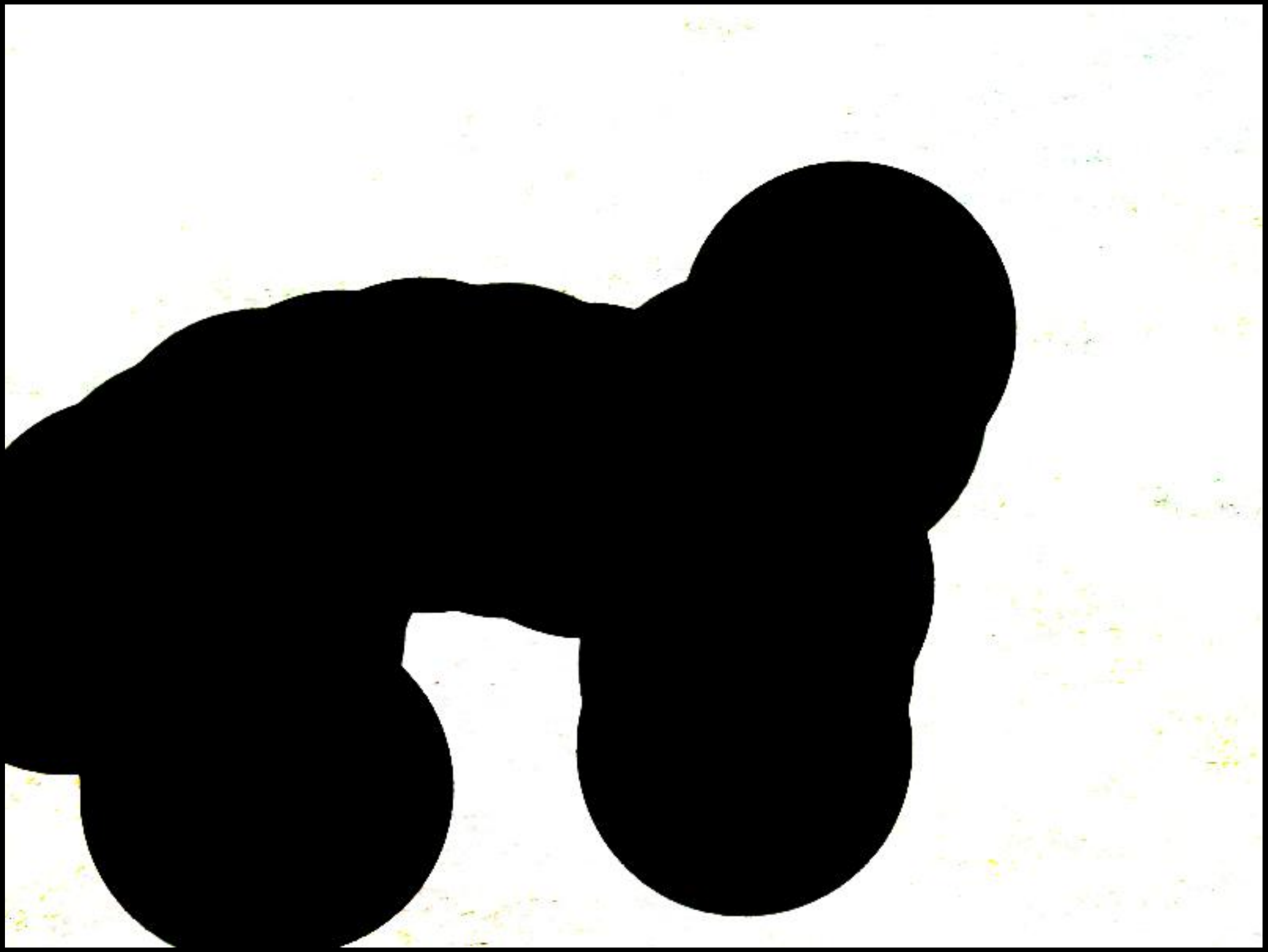
Weight:

Result: Like a max filter but faster

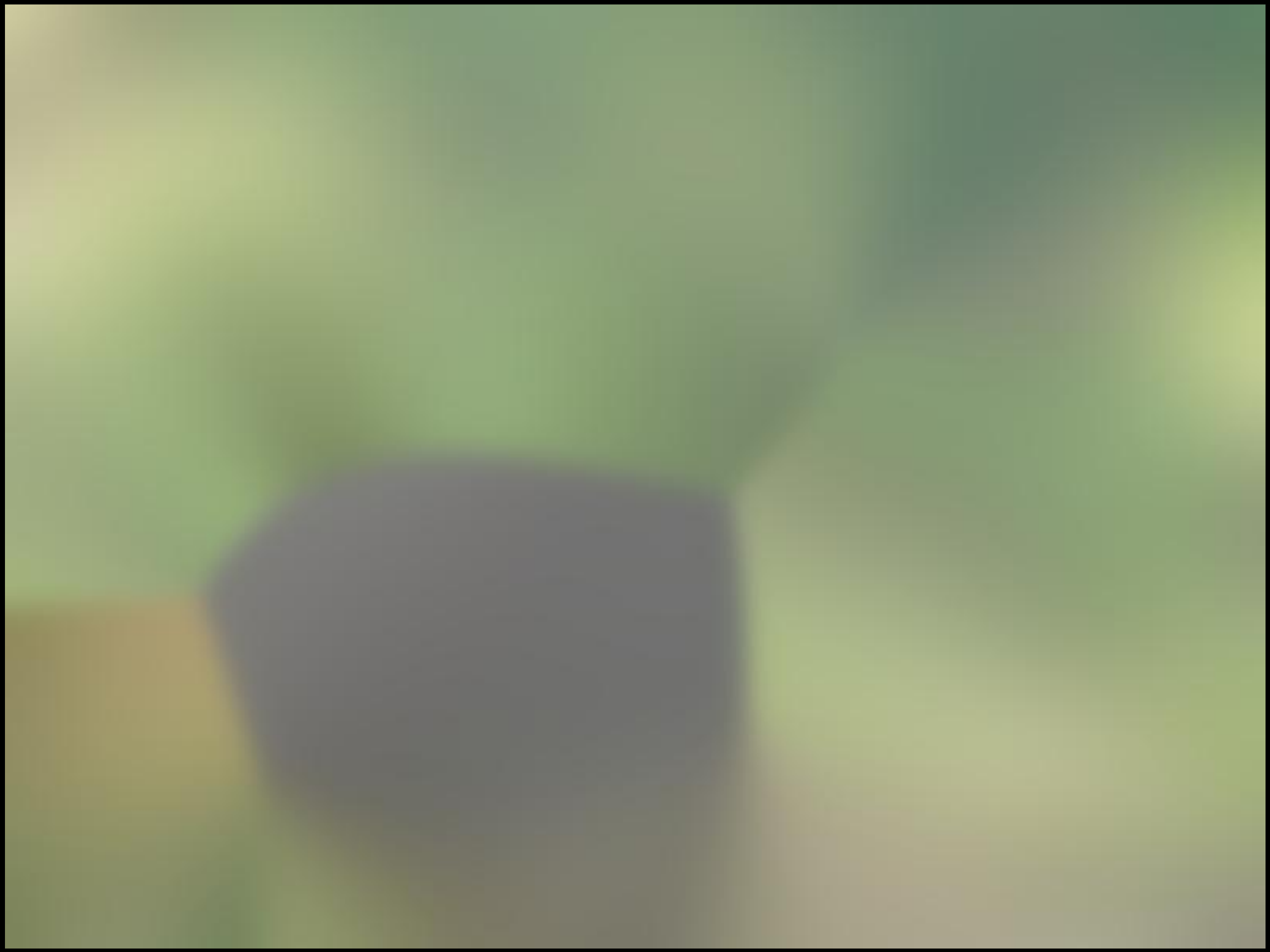
Weight:

Result: Like a min filter but faster

Weight:



Result: A blur that ignores the dog



In ImageStack:

- Convert to homogeneous coordinates:
  - `ImageStack -load dog1.jpg -load mask.png -multiply -load mask.png -adjoin c ...`
- Perform the blur
  - `... -gaussianblur 4 ...`
- Convert back to regular coordinates
  - `... -evalchannels "[0]/[3]" "[1]/[3]" "[2]/[3]" -save output.png`

The Bilateral Filter

- Pixels are mixed with nearby pixels that have a similar value

$$O(x) = \sum_{x'=x-f}^{x+f} I(x').e^{-(\sigma_1(I(x)-I(x'))^2)}.e^{-(\sigma_2(x-x')^2)}$$

- Is this a weighted blur?

$$w(x) = e^{-(\sigma_1(I(x)-I(x'))^2)}$$

The Bilateral Filter

$$O(x) = \sum_{x'=x-f}^{x+f} I(x').e^{-(\sigma_1 (I(x)-I(x'))^2)}.e^{-(\sigma_2 (x-x')^2)}$$

- No, there's no single weight per pixel ☹
- What if we picked a fixed intensity level a, and computed:

$$O(x) = \sum_{x'=x-f}^{x+f} I(x').e^{-(\sigma_1 (a-I(x'))^2)}.e^{-(\sigma_2 (x-x')^2)}$$

The Bilateral Filter

$$O(x) = \sum_{x'=x-f}^{x+f} I(x').e^{-(\sigma_1(a-I(x'))^2)}.e^{-(\sigma_2(x-x')^2)}$$

- This formula is correct when I(x) = a
- And is just a weighted blur, where the weight is:

$$w(x') = e^{-(\sigma_1(a-I(x'))^2)}$$

The Bilateral Filter

- So we have a formula that only works for pixel values close to a
- How can we extend it to work for all pixel values?

The Bilateral Filter

- 1) Pick lots of values of *a*
- 2) Do a weighted blur at each value
- 3) Each output pixel takes its value from the blur with the closest *a*
  - or interpolate between the nearest 2 *a*’s
- Fast Bilateral Filtering for the Display of High-Dynamic-Range Images
  - Durand and Dorsey 2002
  - Used an FFT to do the blur for each value of a

The Bilateral Filter

- Here's a better way to think of it:
- We can combine the exponential terms...

$$O(x) = \sum_{x'=x-f}^{x+f} I(x').e^{-(\sigma_1 (a - I(x'))^2)}.e^{-(\sigma_2 (x-x')^2)}$$

$$\downarrow$$

$$O(x) = \sum_{x'=x-f}^{x+f} I(x').e^{-(\sigma_1 (I(x) - I(x'))^2 + \sigma_2 (x-x')^2)}$$

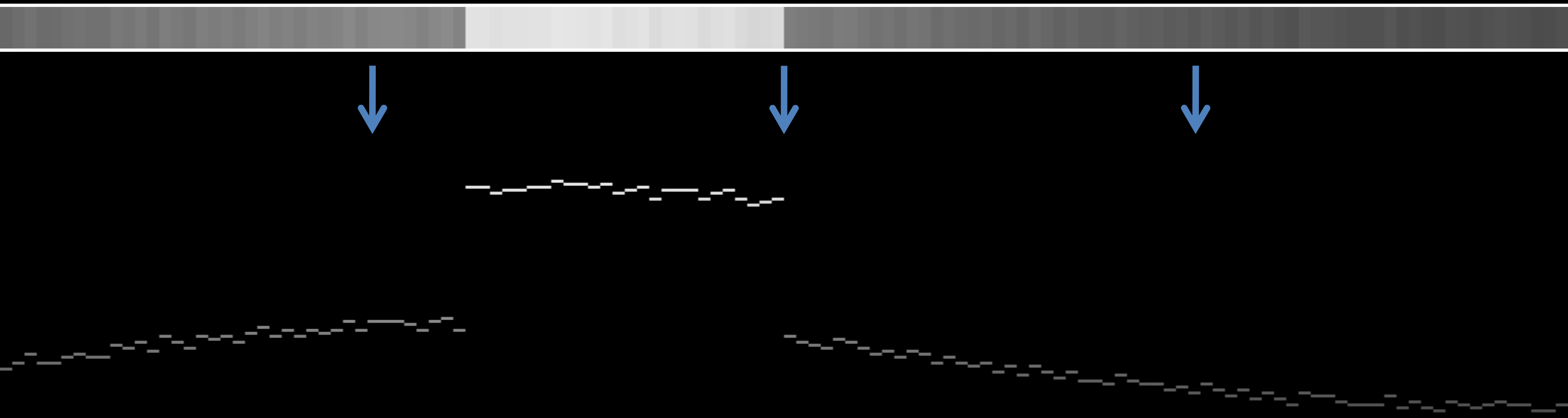
Linearizing the Bilateral Filter

- The product of an 1D gaussian and an 2D gaussian across different dimensions is a single 3D gaussian.
- So we're just doing a weighted 3D blur
- Axes are:
  - image x coordinate
  - image y coordinate
  - pixel value

The Bilateral Grid – Step 1

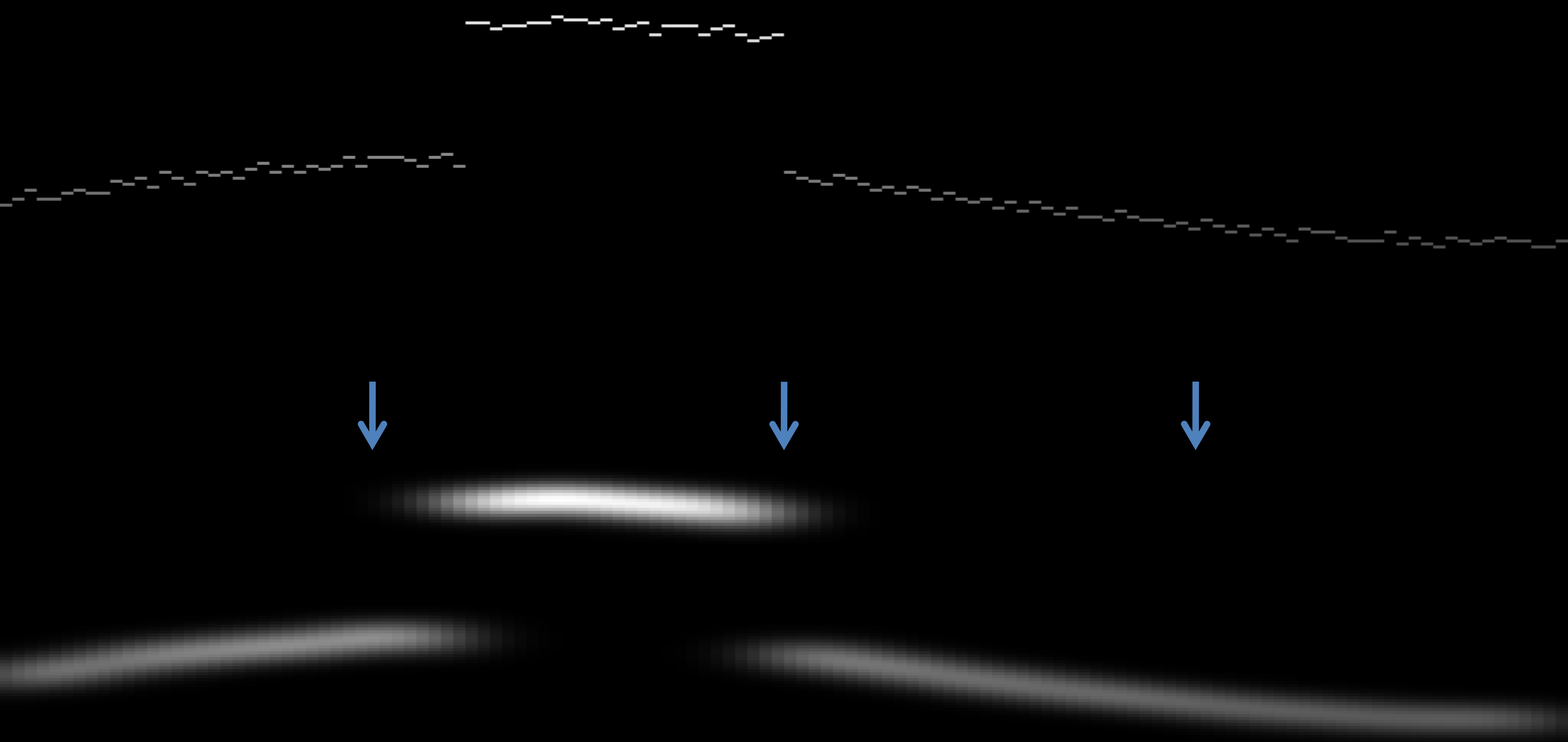
Chen et al SIGGRAPH 07

- Take the 2D image Im(**x**, **y**)
- Create a 3D volume V(**x**, **y**, **z**), such that:
  - Where Im(x, y) = z, V(**x**, **y**, **z**) = **(z, 1)**
  - Elsewhere, V(**x**, **y**, **z**) = (0, 0)



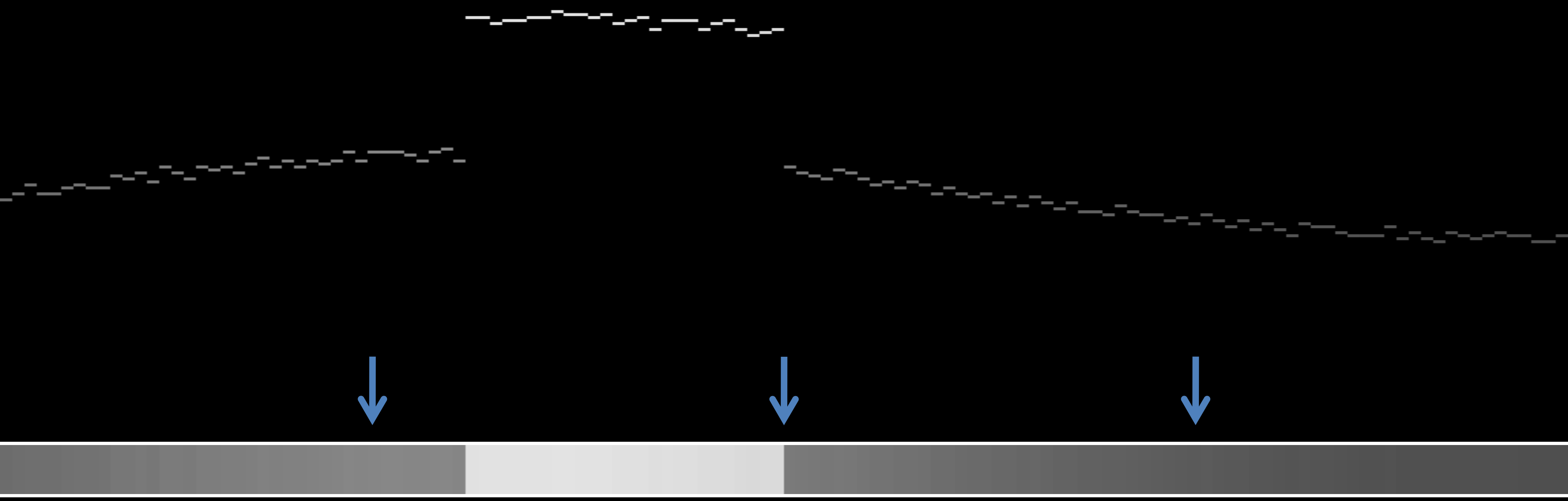
The Bilateral Grid – Step 2

- Blur the 3D volume (using a fast blur)



The Bilateral Grid – Step 3

- Slice the volume at z values corresponding to the original pixel values



Comparison

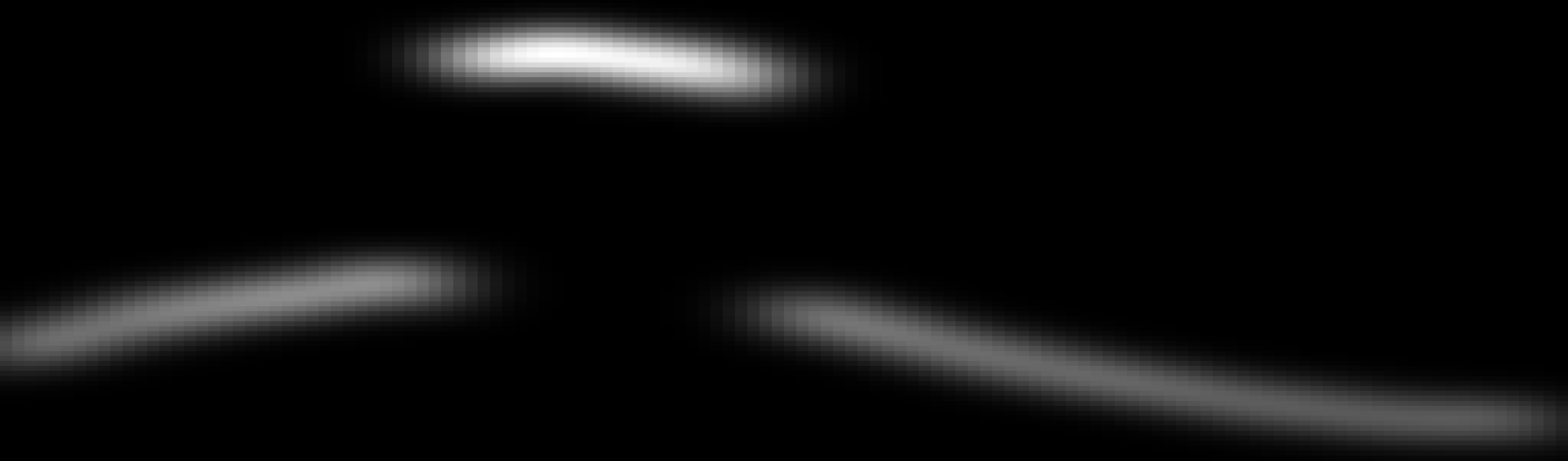
Input

Regular blur

Bilateral Grid Slice

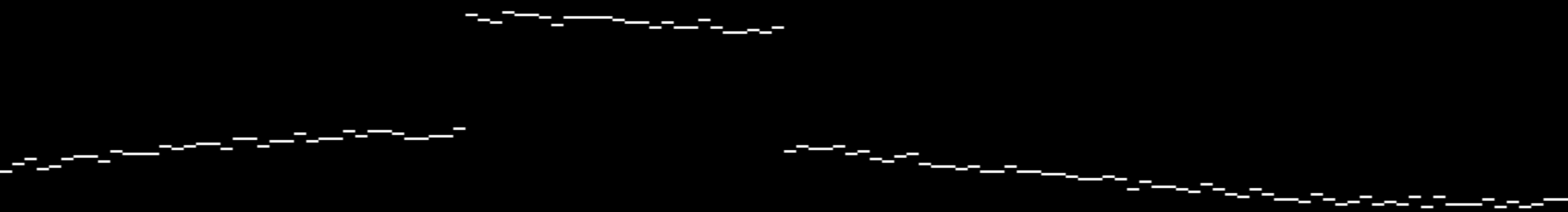
Pixel Influence

- Each pixel blurred together with
  - those nearby in space (x coord on this graph)
  - and value (y coord on this graph)



Bilateral Grid = Local Histogram Transform

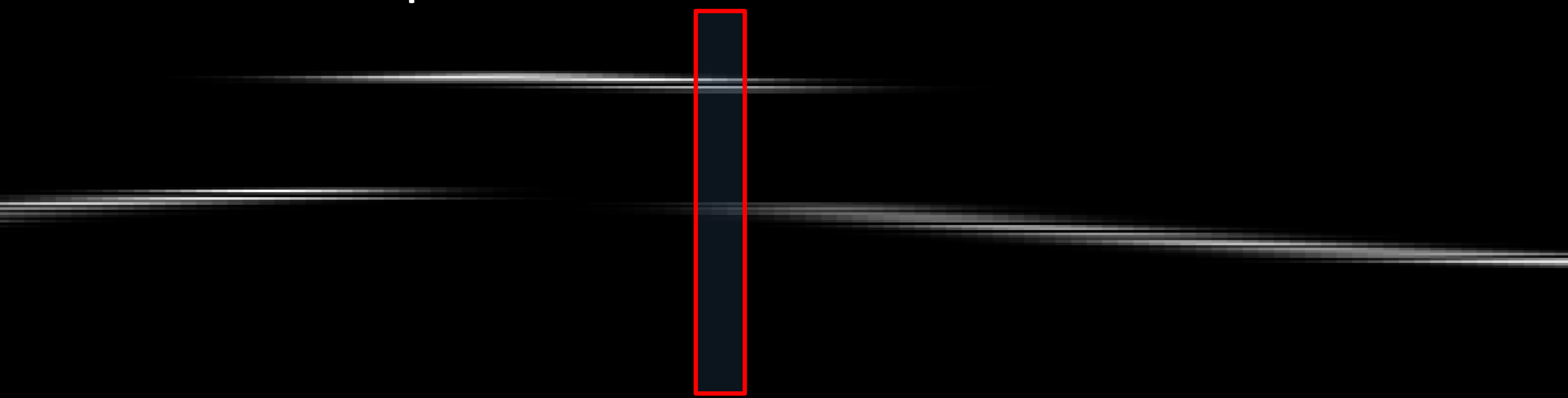
- Take the weight channel:



- Blur in space (but not value)

Bilateral Grid = Local Histogram Transform

- One column is now the histogram of a region around a pixel!



- If we blur in value too, it's just a histogram with fewer buckets
- Useful for median, min, max filters as well.

The Elephant in the Room

- Why hasn't anyone done this before?

- For a 5 megapixel image at 3 bytes per pixel, the bilateral grid with 256 value buckets would take up:
  - 5*1024*1024*(3+1)*256 = 5120 Megabytes

- But wait, we never need the original grid, just the original grid blurred...

Use Filtering by Resampling!

- Construct the bilateral grid at low resolution
  - Use a good downsampling filter to put values in the grid
  - Blur the grid with a small kernel (eg 5x5)
  - Use a good upsampling filter to slice the grid
- Complexity?
  - Regular bilateral filter: O(w*h*f*f)
  - Bilateral grid implementation:
    - time: O(w*h)
    - memory: O(w/f * h/f * 256/g)

Use Filtering by Resampling!

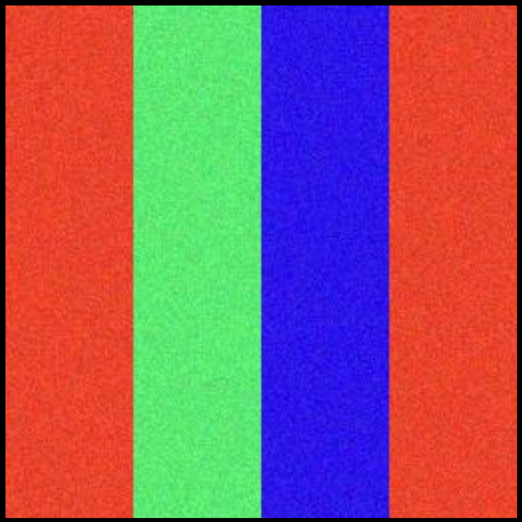
- A Fast Approximation of the Bilateral Filter using a Signal Processing Approach
  - Paris and Durand 2006

Dealing with Color

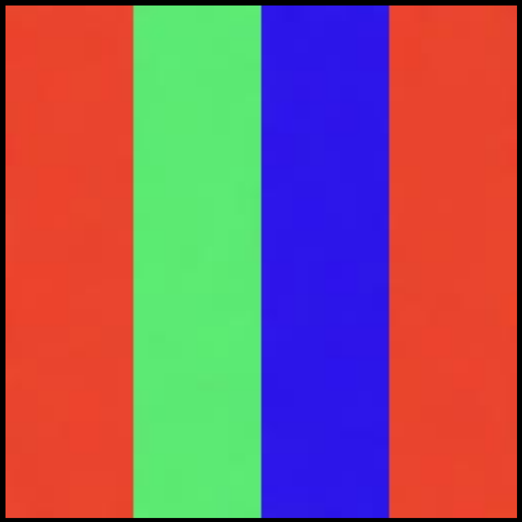
- I’ve treated value as 1D, it’s really 3D
- The bilateral grid should hence really be 5D
- Memory usage starts to go up...
- Cost of splatting and slicing = 2^d
- Most people just use distance in luminance instead of full 3D distance
  - ***values*** in grid are 3D colors (4 bytes per entry)
  - ***positions*** of values is just the 1D luminance = (R+G+B)/3

Bilateral Grid Demo and Video

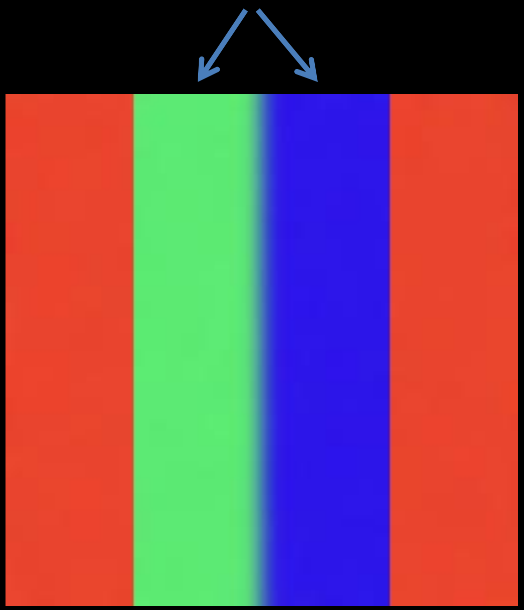
Using distance in 3D
vs
Just using distance in luminance



Input



Full Bilateral



Luminance Only Bilateral

There is a disconnect between positions and values

- **Values** in the bilateral grid are the things we want to blur

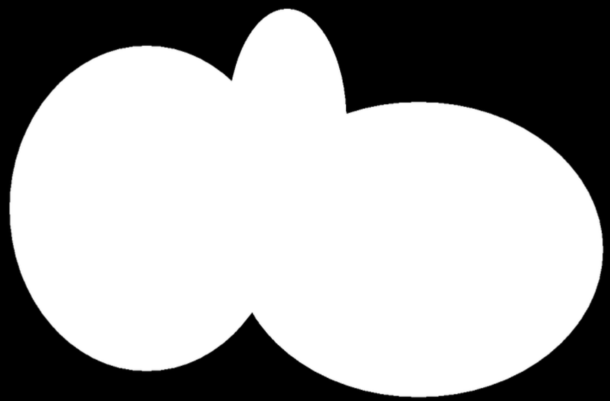
- **Positions** (and hence distances) in the bilateral grid determine which values we mix

- So we could, for example, get the positions from one image, and the values from another

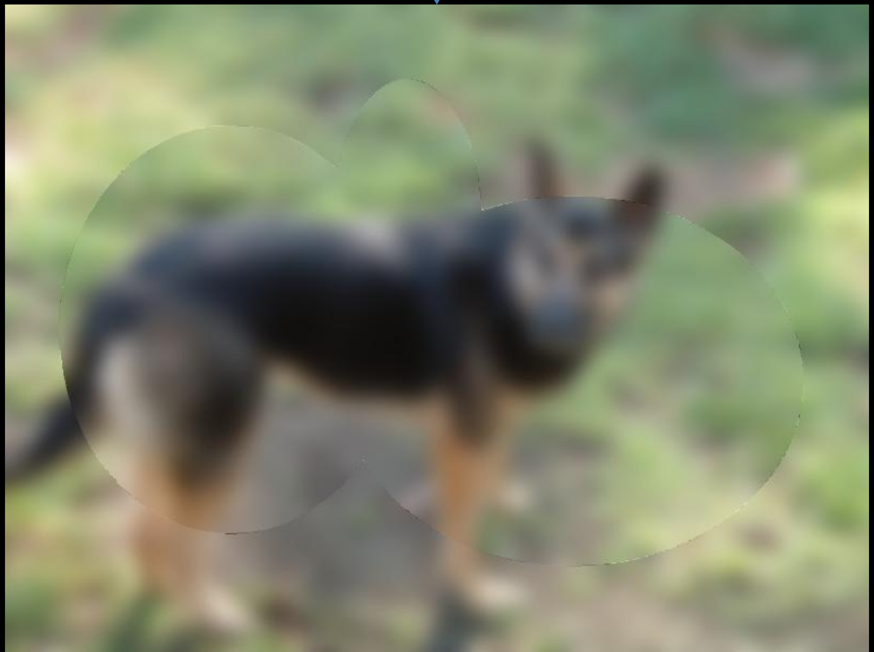
Joint Bilateral Filter

Input Image

Reference Image



Result



Joint Bilateral Application

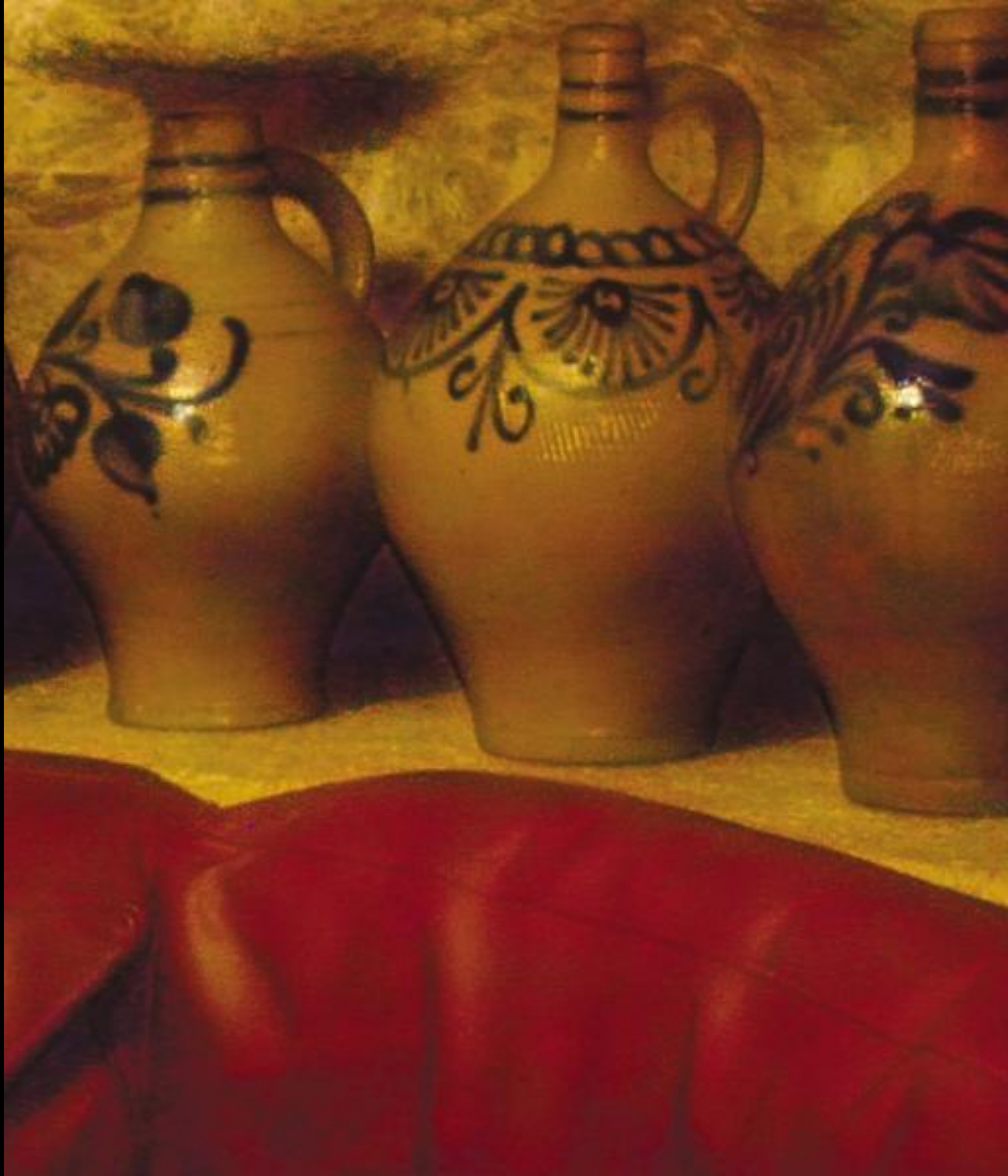
- Flash/No Flash photography
- Take a photo with flash (colors look bad)
- Take a photo without flash (noisy)
- Use the edges from the flash photo to help smooth the blurry photo
- Then add back in the high frequencies from the flash photo
- **Digital Photography with Flash and No-Flash Image Pairs**

  Petschnigg et al, SIGGRAPH 04

Flash:

No Flash:

Result:



Joint Bilateral Upsample

Kopf et al, SIGRAPH 07

- Say we’ve computed something expensive at low resolution (eg tonemapping, or depth)
- We want to use the result at the original resolution
- Use the original image as the positions
- Use the low res solution as the values
- Since the bilateral grid is low resolution anyway, just:
  - read in the low res values at positions given by the downsampled high res image
  - slice using the high res image

Joint Bilateral Upsample Example

- Low resolution depth, high resolution color
- Depth edges probably occur at color edges

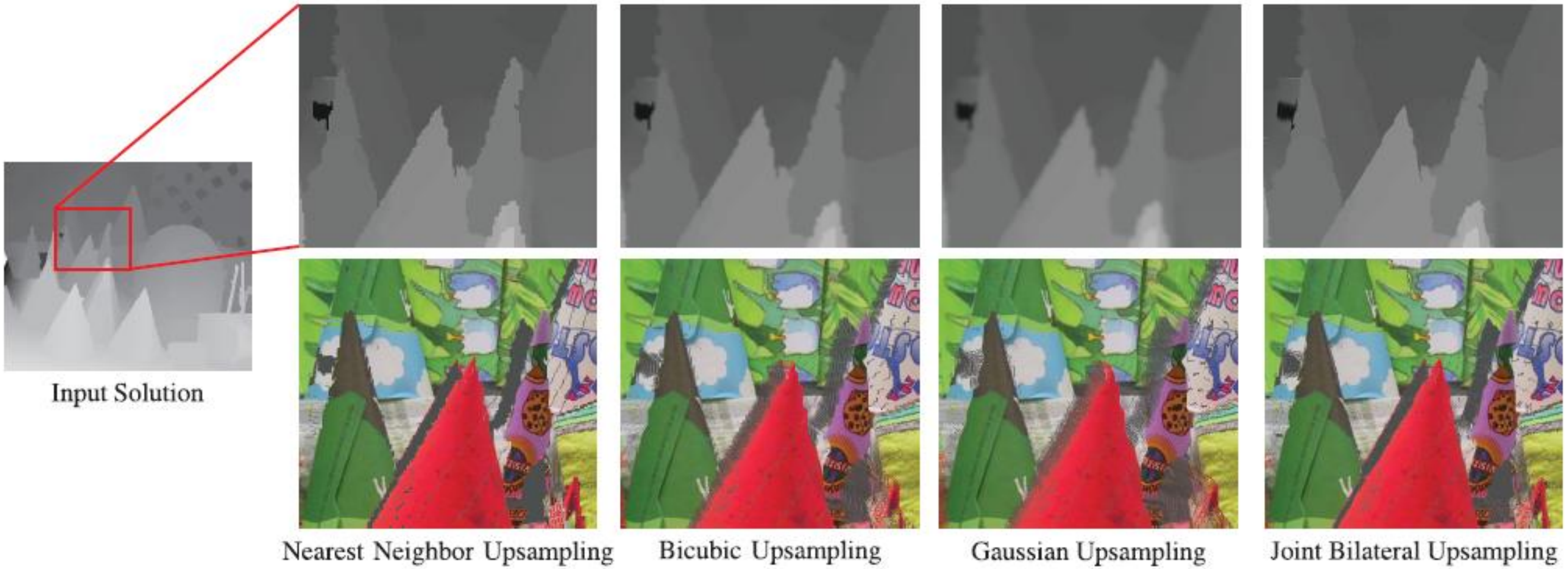


Figure 4: Stereo Depth: The low resolution depth map is shown at left. The top right row shows details from the upsampled maps using different methods. Below each detail image is a corresponding 3d view from an offset camera using the upsampled depth map.

Non-Local Means

- Average each pixel with other pixels that have similar local neighborhoods
- Slow as hell

Think of it this way:

- Blur pixels with other pixels that are nearby in patch-space
- Can use a bilateral grid!
  - Except dimensionality too high
  - Not enough memory
  - Splatting and Slicing too costly (2^d)
- Solution: Use a different data structure to represent blurry high-D space
- (video)

Key Ideas

- Filtering (even bilateral filtering) is O(w*h)
- You can also filter by downsampling, possibly blurring a little, then upsampling
- The bilateral grid is a local histogram transform that's useful for many things